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Crystal Parameter Measurements Simplified

The author describes a procedure to make very accurate measurements on quartz crystals. You can do this with a simple fixture using four resistors, a capacitor and some RF connectors.

This is a technique I derived in 1961 for measuring crystal parameters in a laboratory as an undergraduate student. Fifty years later as radio amateurs, we have much better equipment available on our workbench to do this.

Besides the fixture, the additional equipment needed consists of:

- A digital RF signal generator.
- A frequency counter.
- An RF voltmeter or RF probe.

Crystal Parameters

The quartz crystal unit, in an HC-49U package, consists of a circular quartz disc with aluminum or gold plating on opposite surfaces. The crystal is mounted vertically inside the case. It is held by two supports on the edges of the crystal. Two leads exit the base to secure the crystal in a circuit. Figure 1 is a photo of the internal structure of an HC-49U crystal unit on the left, and the unit in the case on the right.

The quartz crystal unit is electrically represented by a series resistor, R, an inductor, L, and a capacitor, C. A parallel capacitor, C_0, is needed because of the plating and the leads. Figure 2 is the equivalent circuit diagram.

The parameters R, L, and C are referred to in technical publications and books as R_m, L_m, and C_m. The inductance, capacitance and resistance are referred to as the motional parameters of the quartz crystal, thus the subscript *m*.

Derivation of the Resonant Frequency Formulas

The admittance, Y_{AB}, between the terminals A and B in the schematic of Figure 2 is given by Equation 1.

$$Y_{AB} = \frac{1}{Z_{AB}} = \frac{1}{R + j(\omega L - 1/\omega C)} + j\omega C_0 \qquad \text{[Eq 1]}$$

where $\omega = 2\pi f$.

By combining the two terms on the right, we get Equation 2.

$$Y_{AB} = \frac{(1 - \omega^2 LC_0 + C_0 / C) + j\omega RC_0}{R + j(\omega L - 1/\omega C)} \qquad \text{[Eq 2]}$$

and inverting both sides gives us Equation 3.

$$Z_{AB} = \frac{R + j(\omega L - 1/\omega C)}{(1 - \omega^2 LC_0 + C_0 / C) + j\omega RC_0} \qquad \text{[Eq 3]}$$

Multiplying both the numerator and the denominator by the complex conjugate of the denominator gives us Equation 4.

Figure 1 — The internal structure of a quartz crystal unit is shown on the left. The complete package in the metal HC-49U case is shown on the right.

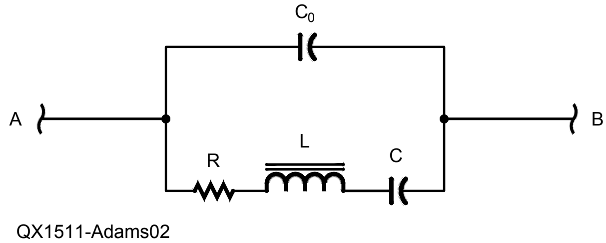

Figure 2 — This is an equivalent circuit for a quartz crystal unit.

$$Z_{AB} = \frac{Real + j(\omega L - 1/\omega C - \omega^3 L^2 C_0 + 2\omega L C_0 / C - C_0 / \omega C^2 - \omega R^2 C_0)}{C_0^2 / C^2 + 1 - 2\omega^2 L C_0 + 2C_0 / C + \omega^2 R^2 C_0^2 - 2\omega L C_0^2 / C + \omega^4 L^2 C_0^2} \quad \text{[Eq 4]}$$

where *Real* is a real number with too many terms to fit on the line. We are not going to use it anyway.

At resonance, the complex component of the above equation is zero. That is the term following the *j*. We can simplify Equation 4 by expressing the complex component as Equation 5.

$$\omega L - 1/\omega C - \omega^3 L^2 C_0 + 2\omega L C_0 / C - C_0 / \omega C^2 - \omega R^2 C_0 = 0 \quad \text{[Eq 5]}$$

Multiplying both sides of the equation by ωC^2 to remove the fractions gives us Equation 6.

$$\omega^2 L C^2 - C - \omega^4 L^2 C^2 C_0 + 2\omega^2 L C C_0 - C_0 - \omega^2 R^2 C^2 C_0 = 0 \quad \text{[Eq 6]}$$

The last term is much smaller than the other terms combined, so we eliminate it. The result is given in Equation 7.

$$\omega^4 L^2 C^2 C_0 - \omega^2 (L C^2 + 2 L C C_0) + (C + C_0) = 0 \quad \text{[Eq 7]}$$

We can solve this equation by finding the roots of the quadratic equation with ω^2 as the independent variable. There are any number of good mathematical software packages that can do this easily. *Wolfram Alpha* is a free online calculator.[1]

There are two resulting resonant frequencies. The series resonant frequency, f_s, is given by Equation 8.

$$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \text{[Eq 8]}$$

The parallel resonant, or antiresonant frequency, f_a, is given by Equation 9.

$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{LC} + \frac{1}{LC_0}} \quad \text{[Eq 9]}$$

We can see that f_a is always greater than f_s.

The crystal is always connected to an external circuit, and C_0 has additional capacitance in parallel with it. We will call that additional capacitance C_p. This will modify Equation 9, and the parallel resonant frequency will be given by Equation 10.

$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{LC} + \frac{1}{LC_t}} \quad \text{[Eq 10]}$$

where $C_t = C_0 + C_p$. Crystal manufacturers specify the resonant frequency of a crystal at this frequency with a particular load capacitance, C_p.

Impedance at Resonant Frequencies

At the series resonant frequency, f_s, we get $\omega_s^2 = 1/LC$, and by plugging this expression into Equation 3 we get Equation 11.

$$Z_{AB} = \frac{R}{1 - (\omega R C_0)^2} \approx R \quad \text{[Eq 11]}$$

We use the approximation because R is on the order of 10 to 100 Ω, and ω is on the order of 10^6, but C_0 is just a few picofarads, and on the order of 10^{-12}. This makes the term very small compared to 1.

For the the parallel or antiresonant frequency we have Equation 12.

$$\omega_a^2 = \frac{1}{LC} + \frac{1}{LC_t} \quad \text{[Eq 12]}$$

Substituting the ω^2 value into Equation 3, and using the impedance of the capacitor, X_C, we obtain Equation 13.

$$Z_{AB} = \frac{1}{\omega^2 C_t^2 R} = \frac{X_c^2}{R} \quad \text{[Eq 13]}$$

The impedance for the parallel or antiresonant frequency is also pure resistance and much greater than the series resonant impedance, with a value typically between 100 kΩ and 1 MΩ.

In order to obtain C_m and L_m, we need only to measure the series resonant frequency and the antiresonant frequency, and the capacitance, C_0. We then use the numbers in Equations 8 and 10 to solve for L_m and C_m. We need a stable and accurate signal generator, an accurate and precise frequency counter and a fairly sensitive RF voltmeter or RF probe. The frequency counter should be able to measure and display frequencies to within 1 Hz. The frequency counter may be built into the signal generator.

The output level from the test fixture at the parallel resonant point is going to be down as much as 110 dB from the peak voltage. This makes this measurement very difficult. Let's find an easier way.

Crystal in Series With A Capacitor

Let's examine a crystal in series with a capacitor. Figure 3 shows the schematic for this model.

The impedance between terminals A and B of the circuit is given by Equation 14.

$$Z_{AB} = \frac{R + j(\omega L - 1/\omega C)}{1 - \omega^2 L C_0 + C_0 / C + j\omega R C_0} + \frac{1}{j\omega C_X} \quad \text{[Eq 14]}$$

We wade through some lengthy arithmetic to find the two resonant frequencies. This is more tedious than the previous derivation, resulting in an expression with more than 20 terms. I will not bore you with the details and leave it as an exercise, if you are interested in a challenge. The two resulting resonant frequencies are given by Equations 15 and 16.

$$\omega_c = \sqrt{\frac{1}{LC} + \frac{1}{LC_t}} \quad \text{[Eq 15]}$$

$$\omega_a = \sqrt{\frac{1}{LC} + \frac{1}{LC_0}} \quad \text{[Eq 16]}$$

where $C_t = C_0 + C_X$ and $\omega = 2\pi f$.

We have shifted the previous series resonant point, now represented as ω_c, up in frequency. The antiresonant frequency remains exactly the same.

We now use Equations 8 and 15 to determine L_m and C_m of the crystal. This is a system of two equations with three unknowns. We measure C_0 directly with an L/C meter. Take Equation 8 and rewrite it as Equation 17.

$$\omega_s^2 = \frac{1}{LC} \quad \text{[Eq 17]}$$

[1]Notes appear on page 26

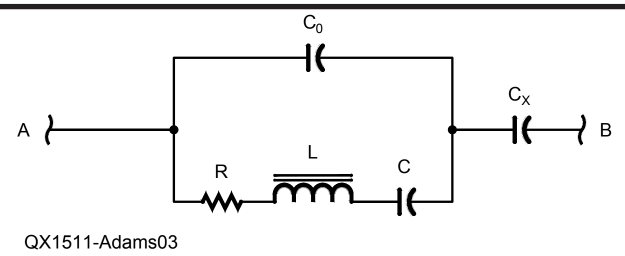

Figure 3 — This schematic diagram is the model circuit for a crystal in series with capacitor C_X.

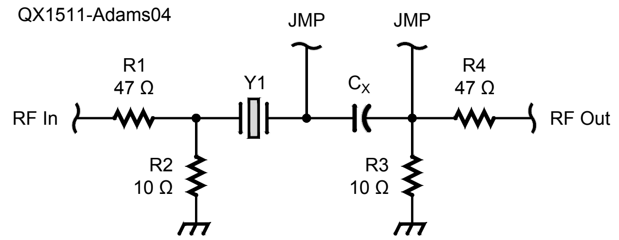

Figure 4 — Here is the schematic diagram for the author's crystal test fixture. R1 = R4 = 47 Ω, R2 = R3 = 10 Ω, Y1 is the crystal under test, and C_X = 47 pF. JMP is a jumper to short out C_X.

We will also rewrite Equation 15 as Equation 18.

$$\omega_c^2 = \frac{1}{LC} + \frac{1}{LC_t} \qquad \text{[Eq 18]}$$

The subscript c indicates that this is a measurement with C_X in series with the crystal.

Subtracting Equation 17 from Equation 18 gives us Equation 19.

$$\omega_c^2 - \omega_s^2 = \frac{1}{LC_t} \qquad \text{[Eq 19]}$$

This now becomes Equation 20.

$$4\pi^2 (f_c^2 - f_s^2) = \frac{1}{LC_t} \qquad \text{[Eq 20]}$$

At this point, everyone wants to make an approximation for the difference of the two squares. Let's use the equation $x^2 - y^2 = (x + y)(x - y)$ and get more precise results. This will give us Equation 21, solved for L_m.

$$L_m = \frac{1}{4\pi^2 (f_c + f_s)(f_c - f_s)(C_0 + C_X)} \qquad \text{[Eq 21]}$$

We can measure the two resonant frequencies using a signal generator and frequency counter. Measure C_0 and C_X using a capacitance meter, and then crunch the numbers.

Test Fixture

In order to make the measurements we use a test fixture. Other test measurements in publications and on the Internet use more complex circuits. This test circuit is very simple. Figure 4 shows the schematic diagram for the circuit. You can see how simple and inexpensive it can be.

The input and output impedance of the fixture is close to 50 Ω, but is not critical. R2 and R3 should be kept small to reduce the loaded *Q* on the crystal, but not too small to attenuate the output RF voltage of the fixture to a very small value. The resonant frequencies are not affected by these values. If the values are large, the resonant peak spreads out and it is more difficult to home in on the exact peak. The small values of R2 and R3 also serve to swamp any effects of stray capacitance in the fixture.

Figure 5 is a photograph of the test fixture that I use for this procedure.

Here are the steps to measure the data needed.

1) With the RF generator voltage connected to the fixture, find the series resonant frequency with capacitor C_X shorted. Start the frequency generator a few kilohertz below the marked frequency of the crystal and slowly increase the frequency while watching the RF output level. Write down the frequency at which the peak output voltage occurs. This is f_s.

2) Remove the short across capacitor C_X. Find the new output voltage peak at a slightly higher frequency. This will be f_c.

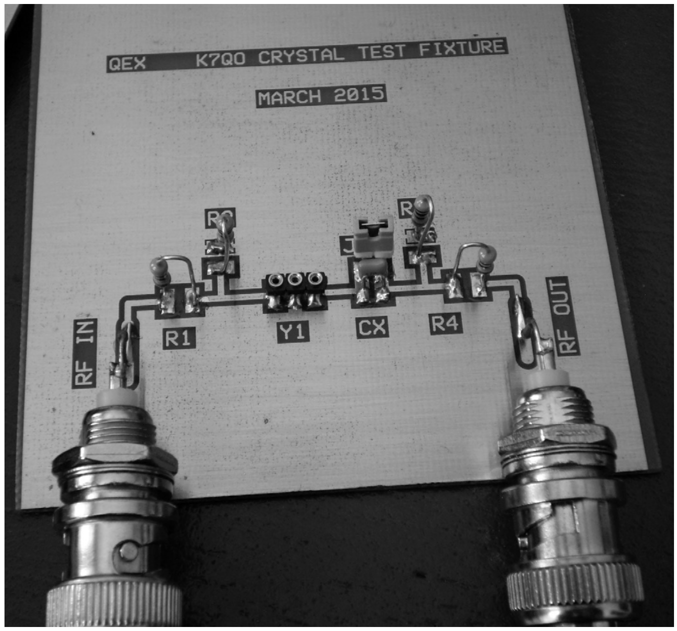

Figure 5 — This photo shows the author's crystal test fixture. It was built using a circuit board layout with parts labeled. The center pin of the crystal socket is grounded to reduce the socket capacitance across the crystal.

3) When you build the fixture measure C_X before installing it and again in the circuit without a crystal in the socket to determine the extra stray capacitance caused by the shorting terminals. I used a 47 pF disc capacitor, but any value near this should do nicely. I recommend an NPØ capacitor.

4) Measure C_0 of the crystal using an accurate L/C meter, such as the Almost All Digital Electronics (AADE) L/C Meter II.[2]

Now you have all the data needed to calculate L_m. We obtain C_m from the series resonant frequency, Equation 8, by using the L_m value and f_s.

Crystal Motional Resistance

Here are the steps to measure the motional resistance, R_m, or the effective series resistance (ESR) of the crystal.

1) Short C_X and again find the series resonant frequency. Make note of the output voltage as accurately as possible.

2) Remove the crystal, leaving everything else as is.

3) Replace the crystal with a variable resistor of 100 Ω. I use a 25 turn variable resistor that has 0.2 inch lead spacing, to fit the crystal socket. Adjust the resistor for the exact same RF voltage output we had with the crystal in the socket.

4) Remove the variable resistor and measure the resistance. This is R_m.

Congratulations. You have all four crystal parameters. You have L_m, C_m, R_m, and C_0. These values may be used to determine the circuit for a crystal IF filter with a specific bandwidth.

You can determine the quality factor, Q, of the crystal by taking the series resonant frequency, f_s, motional inductance, L_m, and series resistance, R_s, and use Equation 22.

$$Q = \frac{X_L}{R_m} = \frac{\omega_s L_m}{R_m} = \frac{2\pi f_s L_m}{R_m} \quad \text{[Eq 22]}$$

This is the inductive reactance at the resonant frequency divided by the motional resistance, R_m, of the crystal.

I have written some *Python* code to perform the calculations for the parameters of the crystal under test. This code carries out the computations to the full 64 bit precision of the computer processor. My code is available for download from the ARRL *QEX* files web page.[3]

Procedure Verification

In order to verify that this procedure is both useful and accurate I picked at random nine crystals from my collection. I then sent these to Tom Thomson, WØIVJ, in Colorado to measure their characteristics by using an AIM Model 4170 Vector Network Analyzer. He also had Larry Benko, WØQE, do the same measurements with another 4170 VNA. Table 1 shows the results, with their measurements and mine. As you can see, the agreement on the crystal parameters is excellent.

SPICE Simulation

As a check of all my theoretical work, I ran a *SPICE* simulation using *ngspice*. I set up an input RF voltage of 1.00 V and swept a crystal model from 4.190 MHz to 4.210 MHz. The voltage output was plotted in dB to show the null depth.

The important thing to note is that the null, corresponding to the parallel resonant mode, remains at the same frequency, but varies in magnitude. This agrees with the theoretical derivation and resulting formula.

Matching Crystals

Using the technique discussed to match crystals is going to be a long and tedious task. One of the things that I want to demonstrate is how a Colpitts crystal oscillator can be used to match crystals and get excellent results.

Suppose you have a number of crystals and you are looking for four crystals for a four pole crystal filter. You want the crystals to match within 10 Hz of each other. Then, using the oscillator and a frequency counter you plug the crystals in and measure the output frequency of each, and keep them ordered. Also note the output voltage from the oscillator. If we have two crystals with the same frequency, we will take the one with the higher output from the oscillator because it will have the lowest R_m.

I have a few hundred 4.096 MHz crystals that I won at an auction on eBay. I found four of them that matched within 5 Hz of each other in the oscillator. I then used the procedure outlined in this paper to measure their crystal parameters. My results are given in Table 2.

Depending upon what program you use to generate the component values for your filters, you can match crystals using the Colpitts crystal oscillator, and then measure the parameters of just one crystal for use in the program. You could also measure a few of the matched crystals and average their parameter values to use in the program. Experimentation will determine which is the fastest method and just how well it meets your criteria for the resulting filter(s).

Conclusion

You now know how to measure crystal parameters accurately and how to easily match a set of crystals for a filter. The test fixture is simple and easy to contruct using any of a number of building techniques. I hope that you will find this test fixture and procedure to be a useful addition to your workbench, and that it will simplify the construction of many successful projects.

Chuck Adams, K7QO, was first licensed as KN5FJZ in the mid 1950s, during the greatest sunspot cycle in recorded history. He has held the calls K5FJZ, K5FO, and now K7QO. He is a retired professor of computer sciences, electrical engineering, and physics. He holds a PhD in physics, with a specialization in radiative transfer and electromagnetics. He now spends his time experimenting and building his own equipment. From time to time, he even gets on the air.

Notes

[1]*Wolfram Alpha* is a free online calculator that will calculate a large variety of quantities from your input values. Go to **www.wolframalpha.com**.

[2]The Almost All Digital Electronics website has had information about an array of kits, including the L/C Meter II at **www.aade.com**. [Unfortunately, when I checked this link prior to publication, the website home page has a note informing us that Neil Heckt passed away on August 19, 2015. The note further indicates that we should be patient while his family determines the future of the company. — *Ed.*]

[3]The author's *Python* code for computing the crystal parameters from the measured data is available for download from the ARRL *QEX* files web page. Go to **www.arrl.org/qexfiles** and look for the file **1x16_Adams.Zip**

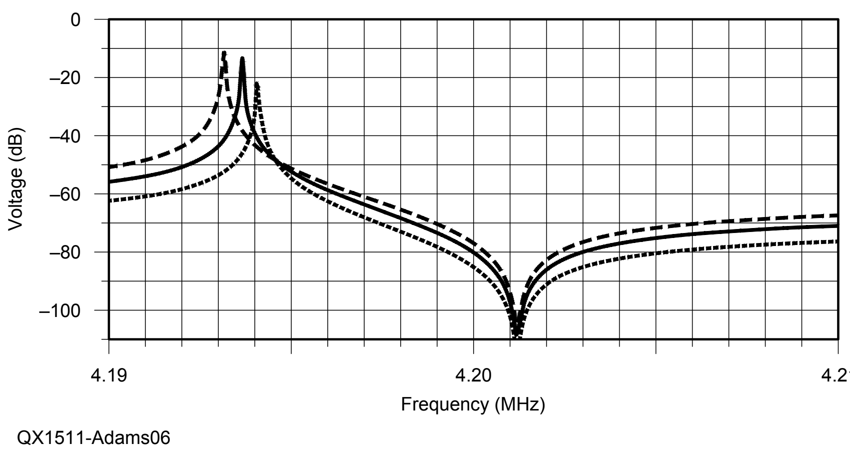

Figure 6 — SPICE simulation for sweeping a crystal. The left-most curve is with no series capacitor and then two more curves for two different values for C_X.

Table 1
Crystal Measurement Procedure Verification

Lab Tech	*Crystal Number*	F_{Series}	$F_{Parallel}$	R_s	L_s (mH)	C_s (pf)	C_p (pf)	Q_s	*Measuring Instrument*
WØIVJ	1	3.578426	3.585154	49.822	139.418	0.0141885	3.780	65801	AIM 4170 VNA
WØQE	1	3.578427	3.585256	49.700	142.449	0.0138866	3.638	64443	AIM 4170 VNA
K7QO	1	3.578426	--------	49.6	141.624	0.013968	3.65	64199	K7QO Fixture
WØIVJ	2	4.193154	4.200966	16.943	111.185	0.0129572	3.484	180122	AIM 4170 VNA
WØQE	2	4.193163	4.200894	17.159	115.719	0.0124495	3.376	177674	AIM 4170 VNA
K7QO	2	4.193159	--------	15.4	114.234	0.012611	3.22	195432	K7QO Fixture
WØIVJ	3	4.031548	4.036547	40.962	309.640	0.0050332	2.032	211124	AIM 4170 VNA
WØQE	3	4.031552	4.036428	40.046	340.051	0.0045830	1.895	215101	AIM 4170 VNA
K7QO	3	4.031553	--------	39.1	326.544	0.004773	1.57	211552	K7QO Fixture
WØIVJ	4	4.193152	4.201202	18.176	107.122	0.0134487	3.509	165524	AIM 4170 VNA
WØQE	4	4.193157	4.201100	18.432	112.633	0.0127907	3.376	160993	AIM 4170 VNA
K7QO	4	4.193154	--------	17.5	112.514	0.012804	3.44	169390	K7QO Fixture
WØIVJ	5	4.094814	4.102963	23.238	134.404	0.0112398	2.830	147508	AIM 4170 VNA
WØQE	5	4.094819	4.103052	23.469	134.440	0.0112368	2.794	147386	AIM 4170 VNA
K7QO	5	4.094819	--------	21.8	130.161	0.0130161	2.74	153616	K7QO Fixture
WØIVJ	6	3.998939	4.005005	22.484	132.716	0.0119351	3.940	153331	AIM 4170 VNA
WØQE	6	3.998953	4.005015	22.619	136.166	0.0116326	3.837	151261	AIM 4170 VNA
K7QO	6	3.998947	--------	21.6	133.566	0.0133566	3.80	155370	K7QO Fixture
WØIVJ	7	11.055203	11.079818	7.407	11.640	0.0178059	4.007	109648	AIM 4170 VNA
WØQE	7	11.055211	11.079788	7.337	11.755	0.0176319	3.965	111282	AIM 4170 VNA
K7QO	7	11.055188	--------	7.1	11.449	0.018102	3.99	112009	K7QO Fixture
WØIVJ	8	4.094873	4.102956	24.034	130.270	0.0115962	2.943	144651	AIM 4170 VNA
WØQE	8	4.094876	4.103023	24.476	134.745	0.0112110	2.818	141641	AIM 4170 VNA
K7QO	8	4.094880	--------	24.1	132.374	0.011412	2.90	161414	K7QO Fixture
WØIVJ	9	13.499968	13.529170	4.063	5.074	0.0273900	6.345	100390	AIM 4170 VNA
WØQE	9	13.499973	13.529200	4.129	5.064	0.0274465	6.339	104041	AIM 4170 VNA
K7QO	9	13.499920	--------	4.10	4.739	0.029329	6.10	98042	K7QO Fixture

Number	*Form Factor*	*Crystal Identification Printed on Each Unit*
1	HC-49U	MPCO 3.579545
2	HC-49U	HOSONIC 4.1943 B603
3	HC-49S	4.032
4	HC-49U	HOSONIC 4.1943 B603
5	HC-49U	MMD A18BA1 4.096JHz 9942G
6	HC-49U	ABRACON 4.000 AB 0443
7	HC-49U	FOX115-20 11.0592
8	HC-49U	MMD A18BA1 4.096MHz
9	HC-49U	78941-1 13.500 KDS 5K

Table 2
Sample Crystal Measurements

Crystal	f_s (Hz)	f_c (Hz)	L_m (mH)	C_m (fF)	C_0 (pF)
1	4094849	4095292	132.12	11.43	2.97
2	4094849	4095287	133.94	11.28	2.85
3	4094846	4095294	130.82	11.54	2.90
4	4094849	4095301	129.62	11.65	2.92